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Design of Flight Control Systems to Meet Rotorcraft Handling Qualities Specifications

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This paper describes a methodology for the design of control laws for improvement of helicopter handling qualities. The design procedure uses eigenstructure assignment techniques for the design of inner-loop control laws that decouple roll, pitch, and yaw rates and vertical velocity, provide appropriate bandwidths in all channels, and stabilize low-frequency open-loop instabilities. With the inner loops closed, the angular rates and vertical velocity responses to commands are approximated by four decoupled first-order systems. Various response types, such as attitude command attitude hold, can then be easily realized by simple single-loop feedbacks and feedforwards wrapped around these inner loops. Both time and frequency responses show that the closed-loop helicopter provides excellent nominal performance in terms of tracking of pilot commands and achievement of desired response type characteristics. Stability robustness is investigated by unstructured singular value techniques. Performance and stability are also verified by simulations using an off-nominal flight condition, and a higher order model that includes explicit models of rotor and actuator dynamics, and a time delay model of other higher order dynamics and delays due to digital implementation of the control laws. The control laws presented in this paper provide better stability robustness and performance and are simpler in structure than those designed using a previous eigenstructure procedure proposed by the authors.

Nomenclature

Scalars

i = $\sqrt{-1}$
p = roll rate, rad/s
q = pitch rate, rad/s
r = yaw rate, rad/s
s = Laplace operator
u = forward velocity, ft/s
v = lateral velocity, ft/s
w = vertical velocity, ft/s
$\Delta\Phi(2\omega_{180})$ = 180 deg + phase angle at $2\omega_{180}$
θ = pitch angle, rad
λ = eigenvalue
$\underline{\sigma}(A)$ = minimum singular value of matrix A
$\bar{\sigma}(A)$ = maximum singular value of matrix A
τ = effective time delay, s
τ_p = phase delay, s
ϕ = roll angle, rad
ω_{180} = frequency at which the phase angle is -180 deg, rad/s

Vectors

u = control vector, $[\delta_{coll}, \delta_{lat}, \delta_{long}, \delta_{TR}]^T$
x = state vector of rigid body states, $[u, v, w, p, q, r, \phi, \theta]^T$
V_a = achievable nondimensional closed-loop eigenvectors
V_d = desired nondimensional closed-loop eigenvectors

Matrices

A_8 = open-loop dynamics matrix, eighth-order model
A_{12} = open-loop dynamics matrix, 12th-order model
A^d = desired closed-loop dynamics matrix
B_8 = control distribution matrix, eighth-order model
B_{12} = control distribution matrix, 12th-order model
B^d = desired control distribution matrix
C = measurement distribution matrix
$E(s)$ = multiplicative error matrix
$G(s)$ = open-loop transfer matrix, $C(Is - A)^{-1}B$
$G(s)_{\text{closed loop}}$ = transfer matrix with inner loops closed, $C(Is - A + BK)^{-1}BH$
H = feedforward gain matrix
K = feedback gain matrix
$K(s)$ = compensator transfer matrix
$K(s)G(s)$ = loop transfer matrix

Subscripts

bw = bandwidth
c = commanded

Superscripts

d = desired
T = transposed
-1 = inverse

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Introduction

MILITARY handling quality specifications are currently being revised for advanced rotorcraft. The new specifications are expressed in terms of response types.[1] These response types are formulated as desired transfer functions between pilot inputs and vehicle outputs. Three response types have been developed to quantify mission-oriented rotorcraft handling quality specifications.[1,2] These are as follows: 1) attitude command with attitude hold (ACAH), 2) rate command with attitude hold (RCAH), and 3) translational rate command with position hold (TRCPH).

The ACAH response type results in a pitch (roll) attitude angle proportional to the longitudinal (lateral) stick input. When the input is removed, the helicopter returns to the trim attitude that existed before the input was applied. This response type does not have the agility of the RCAH type but is desirable for hover and low-speed operations, particularly in conditions of degraded visual cueing and for divided attention tasks. In the RCAH response type, angular velocities about the vehicle roll, pitch, or yaw axes are proportional to the corresponding cockpit controller input. The RCAH response type has a high degree of agility and is preferred for fully attended operations in conditions of good visual cueing. In the TRCPH response type, a constant cockpit controller input must produce a constant translational rate, and the rotorcraft must hold position if the cockpit controller input is zero. TRCPH is useful for precision hovering tasks and may be necessary to achieve level 1 handling quality in nap-of-the-Earth maneuvers in fair to poor visual cue environments.

Although most unaugmented helicopters exhibit RCAH response, they will not meet the new handling quality specifications. Bandwidths are typically not sufficiently high, substantial coupling exists between responses to vertical, longitudinal, lateral, and directional control inputs, and low-frequency phugoid type instabilities may have unacceptably large doubling times. Feedback control systems are necessary to improve handling qualities so that safe operation close to the Earth in poor weather conditions or at night is possible. Since helicopter responses to control inputs are highly coupled, helicopter dynamics are characterized by multi-input/multi-output (MIMO) mathematical models, and design of flight control systems for such vehicles is a true multivariable synthesis problem. An excellent review of research in the area of multivariable design techniques applied to helicopter flight control system design is given by Manness et al.[3]

One approach to design of helicopter flight control laws that has received recent attention is eigenstructure assignment. Eigenstructure assignment is a technique for synthesis of feedback control laws that allows the designer to place closed-loop eigenvalues in specified locations and shape closed-loop eigenvectors. Eigenstructure assignment techniques provide a direct, simple method for achieving desired response characteristics when performance requirements can be expressed in terms of closed-loop eigenvalues and eigenvectors, as is the case with handling quality specifications. Eigenstructure techniques do not provide any guarantees of either performance or stability robustness, however, and these quantities must be carefully evaluated when eigenstructure techniques are used.

Early work in the use of eigenstructure assignment for helicopter control system design was reported by Parry and Muray-Smith,[4] Garrard and Liebst,[5] and Ekblad.[6] These researchers used eigenstructure assignment for decoupling and stabilization but not for direct achievement of response types. Apkarian[7] also applied eigenstructure assignment to a helicopter to illustrate his approach to robustness enhancement. Innocenti and Stanziola[8] have described performance/robustness tradeoffs for eigenstructure control laws for rotorcraft. In a recent paper, Hughes et al.[9] used eigenstructure assignment to design inner loop control laws that stabilize, simplify, and decouple the responses to control inputs. Simplification is accomplished by selecting eigenvalue/eigenvector pairs to cancel stable invariant zeros and associated left zero vectors. This is analogous to pole-zero cancellation for single-input/single-output (SISO) systems. Decoupling and stabilization are accomplished by suitable choices of the remaining eigenvalue/eigenvector pairs. ACAH command following is then accomplished by using a precompensator designed using the Broussard command generator approach. Samblancat et al.[10] use a combined eigenstructure and H_∞ approach. Inner loop control lows that decouple the response into longitudinal, lateral, and directional modes are designed using eigenstructure assignment. An ACAH response is then achieved using the H_∞ technique. The objective of the decoupling is to reduce the order of the H_∞ compensators without loss of performance.

Table 1 Open-loop eigenvalues of the eighth-order model

$-3.2377+0.0000i$	Roll rate
$0.2110\pm0.5296i$	Forward velocity
$0.0353\pm0.7431i$	Side slip
$-0.9021+0.0000i$	Pitch rate
$-0.5695+0.0000i$	Yaw rate
$-0.3221+0.0000i$	Heave velocity

Garrard et al.[11] use eigenstructure assignment to design control laws that achieve desired response types directly. This is accomplished by using the desired response types to generate state space models, which in turn generate desired sets of eigenvalues/eigenvectors. Eigenstructure assignment is then used to design control laws that give the best least squares approximation of the desired eigenstructure. Control laws designed using this method gave good performance when applied to the nominal design model but exhibited lack of stability robustness to variations in aerodynamic coefficients. Low-frequency instabilities resulted from small variations in some of these coefficients. In addition, the structure of the control system was complex, and implementation of different response types would have involved reconfiguration of all of the feedback gains.

To correct these problems, a somewhat different design approach is described in this paper. The control law structure is divided into inner and outer feedback loops. An eigenstructure methodology is used to design inner loop control laws that decouple the vertical velocity, roll, pitch, and yaw rates and provide rate command response in each control loop. Eigenvalue placement is used to stabilize unstable modes and provide appropriate bandwidth, and eigenvector shaping is used for modal decoupling. With the inner loops closed, the helicopter response to pilot commands appears as four decoupled, first-order systems. Classical SISO techniques are used to design outer loop controllers that give a variety of desired response types.

A modern single rotor, four-bladed attack helicopter operating at low speed and hover is used to illustrate the method. The dynamic response characteristics for the helicopter modeled in this study are typical of most high-performance helicopters. Simulations and flight tests have shown that, with such helicopter dynamics, even experienced helicopter pilots are unable to accomplish divided attention operations or relatively simple tasks in degraded visual environments. Handling qualities that minimize the involvement of the pilot in basic stabilization tasks are required to accomplish such operations. This requires a high-bandwidth flight control system.

High-bandwidth flight control systems require high-bandwidth actuators[12]; hence, the rate and deflection limit characteristics of the actuators impose significant limitations on the feedback gains. Additional factors that limit the maximum feedback gains are 1) sensor noise amplification; 2) in-plane (lead-lag) rotor coupling and inflow dynamics[13]; 3) phase-margin requirements and high-frequency modeling uncertainty (rotor and structural flexure modes); 4) propulsion system dynamics; and 5) the time delay due to digital implementation of the flight control laws.[12] In addition, uncertainties in the aerodynamic coefficients can introduce low-frequency uncertainties that can affect stability.

At present it is not feasible to incorporate these effects directly into the mathematical model used for control law design. Many of these quantities can neither be modeled accurately nor measured easily. In addition, inclusion of these effects results in an extremely high-order model that would be impractical to use in design. In this study the mathematical model used for design is based on a rigid body model of the fuselage dynamics with a residualized tip path plane model of rotor dynamics. This is an appropriate model to use for control law design since the state variables of the model are the quantities to be controlled; however, the effects of modeling

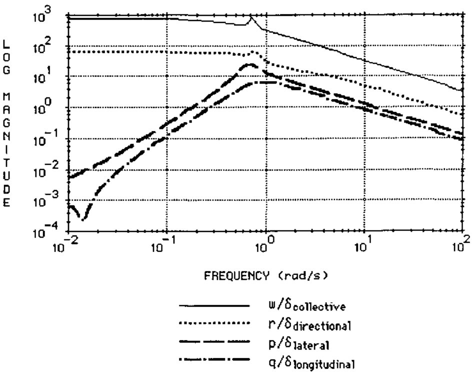

Fig. 1 Singular value plot of open-loop helicopter at hover.

errors on performance and stability must be evaluated. An error model that includes both high- and low-frequency uncertainties is constructed, and the stability robustness of the control laws is evaluated using unstructured singular value analysis. Performance and stability are also evaluated using time simulations of an off-nominal flight condition and a higher order model that contains actuator and rotor dynamics and time delays due to filtering and digital implementation of the control laws.

Desired Response Characteristics

The inner-loop flight control system involves four channels with vertical velocity, roll, pitch, and yaw rates as outputs and collective pitch, cyclic lateral, cyclic longitudinal, and tail rotor collective pitch as inputs. The design of the helicopter is based on an eighth-order model whose states are the rigid body modes of the helicopter. This model was obtained by residualization of a 12th-order model that contained regressing and advancing rotor tip path plane modes. These models are given in the Appendix. The open-loop eigenvalues for the eighth-order model of the helicopter at hover are listed in Table 1. The dominant eigenvalue for each mode is also listed in Table 1, although there is considerable intermodal coupling.

The open-loop helicopter exhibits two undamped unstable modes (forward velocity and side slip) with frequencies of approximately 0.57 and 0.74 rad/s and damping ratios of -0.37 and -0.048, respectively. The doubling times for forward velocity and side slip are 3.27 and 19.5 s, respectively. Level 1 handling qualities specify minimum allowable time to double amplitude for low-frequency second-order modes of 15 s.[14,15] In addition, only the roll bandwidth is sufficiently high to meet level 1 handling qualities requirements.

The open-loop singular values of the 4×4 transfer function matrix relating control inputs to sensor outputs for the helicopter at hover are shown as a function of frequency in Fig. 1. From the singular vector components of each singular value, it is possible to identify the dominant response associated with each input. These dominant responses are indicated in Fig. 1. As would be expected, the dominant responses associated with the lateral and longitudinal cyclic inputs are roll and pitch rates, whereas yaw rate and vertical velocity are associated with directional and collective inputs. The singular values associated with the lateral and longitudinal inputs are very small at low frequencies (0.0006 and 0.006 at a frequency of 0.01 rad/s). Singular values are analogous to loop gains for SISO systems; thus, low-frequency tracking of roll and pitch rate commands will be very poor since the low-frequency singular values are so small.

For rate command, the desired transfer functions between commands and inner loop regulated variables are given by:

$$\frac{w}{w_c} = \frac{\lambda_w}{(s+\lambda_w)} \tag{1}$$

$$\frac{p}{p_c} = \frac{\lambda_p}{(s+\lambda_p)} \tag{2}$$

$$\frac{q}{q_c} = \frac{\lambda_q}{(s+\lambda_q)} \tag{3}$$

$$\frac{r}{r_c} = \frac{\lambda_r}{(s+\lambda_r)} \tag{4}$$

The desired transfer functions between forward velocity to pitch command and side velocity to roll command are

$$\frac{u(s)}{q_c(s)} = \frac{\lambda_q}{s(s+\lambda_u)} \approx \frac{\lambda_q}{s^2} \tag{5}$$

$$\frac{v(s)}{p_c(s)} = \frac{\lambda_p}{s(s+\lambda_v)} \approx \frac{\lambda_p}{s^2} \tag{6}$$

The eigenvalues λ_u and λ_v are associated with the linearized drag forces in the forward and side directions. Since the values of λ_u and λ_v are small, the transfer functions between lateral and longitudinal velocities and roll and pitch commands are essentially $\lambda_{p,q}/s^2$. Equations (1-6) are used to define the desired closed inner-loop eigenstructure for the model. Eigenstructure assignment is then used to design inner-loop feedback control laws that approximate the desired transfer functions given in Eqs. (1-6). Various command response types can be achieved by simple single-loop feedbacks or feedforwards wrapped around the inner loops.

The desired pitch and roll attitude transfer functions for ACAH are[1,2]:

$$\frac{\phi}{\phi_c} = \frac{\omega_\phi^2}{(s^2+2\zeta\omega_\phi s+\omega_\phi^2)} \tag{7}$$

$$\frac{\theta}{\theta_c} = \frac{\omega_\theta^2}{(s^2+2\zeta\omega_\theta s+\omega_\theta^2)} \tag{8}$$

Similar second-order transfer functions are desired for RCAH and TRCPH.

Inner Design Loop via Eigenstructure Assignment

The theory of eigenstructure assignment is discussed in Refs. 16-18. The feedback control law is assumed to be a linear function of the state vector

$$u = -Kx \tag{9}$$

and the feedback gain matrix K is selected such that this control law results in desired placement of the closed-loop eigenvalues and shaping of the corresponding closed-loop eigenvectors. The ideal closed inner-loop state equations for the helicopter model are derived from the desired transfer functions Eqs. (1-6) as:

$$\dot{x} = A^d x + B^d x_c \tag{10}$$

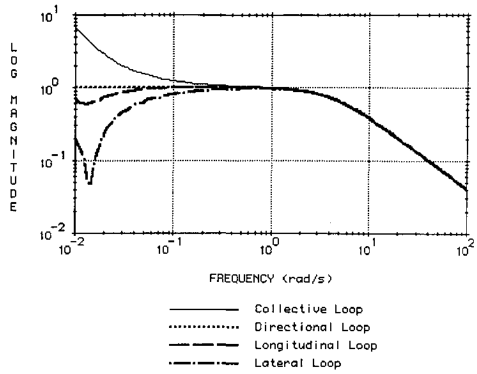

Fig. 2 Singular values of $[G_{\text{closed inner loop}}]$.

where

$$A^d = \begin{bmatrix} -\lambda_u & 0 & 0 & 0 & 1 & 0 & 0 & \lambda_q \\ 0 & -\lambda_v & 0 & 1 & 0 & 0 & \lambda_p & 0 \\ 0 & 0 & -\lambda_w & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda_p & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_r & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B^d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \lambda_w & 0 & 0 & 0 \\ 0 & \lambda_p & 0 & 0 \\ 0 & 0 & \lambda_q & 0 \\ 0 & 0 & 0 & \lambda_r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The values of $\lambda_{w,p,p,r}$ for the ideal A matrix are determined from the handling qualities requirements. A bandwidth of 4 rad/s was chosen for vertical velocity, roll, pitch, and yaw rate. Based on previous experience, the values of λ_u and λ_v were selected to be -0.00199 and -0.00526, 10% of their nominal values.[11] With the closed-loop eigenvalues specified, A^d is then used to define the desired eigenvalue/eigenvector configuration. This configuration is achieved in a best least square sense using eigenstructure assignment. The desired and attainable eigenvalues are given in the Appendix, as is the gain matrix K. The full-state regulator in the inner loop requires that all states be fed back. Since in some cases all of the state variables may not be measurable, it may be necesary to include a state estimator in the feedback loop in order to implement the control law. The details of the eigenstructure design of an estimator that preserves the loop transfer properties of the full state control law are given in Ref. 11.

The extensive coupling of the control distribution matrix B requires a feedforward gain matrix to alleviate the problem of

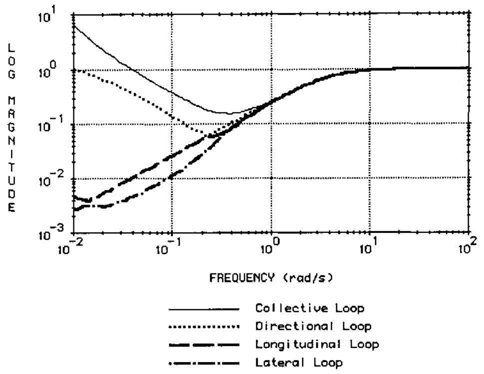

Fig. 3 Singular values of $[I - G_{\text{closed inner loop}}]$.

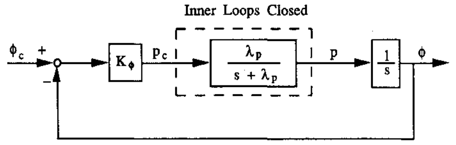

Fig. 4 Block diagram of roll loop.

Table 2 Eigenvalues with outer and inner feedback loops closed

$-0.0006 \pm 0.0140i$	Phugoid
$-2.0107 \pm 1.9866i$	Pitch attitude
$-1.9925 \pm 2.0038i$	Roll attitude
$-4.0000 + 0.0000i$	Yaw rate
$-4.0000 + 0.0000i$	Vertical velocity

Table 3 Summary of time delay contributors

Element	Delay, ms
Rotor	66
Actuators	31
Zero-order hold	17
Computations	22
Notch filter	11
Total	147

control cross-coupling. As in Ref. 11, a static feedforward gain matrix H was selected such that

$$B^*H \approx B^d \tag{11}$$

H is determined using the pseudo-inverse method,[19] which is optimal in the sense that $\text{tr}(B^{T*}B)(B^{T*}B)$ is minimized. Solving for H yields

$$H = (B^{T*}B)^{-1*}B^{T*}B^d \tag{12}$$

This result achieves the decoupling desired for the four control inputs. It also includes some unwanted input in forward and side velocities for heave command, forward velocity for longitudinal command, and side velocity for yaw rate command. Although these coupling terms are undesirable, they are tolerable since their magnitudes are small. The matrix H is given in the Appendix. Static control decoupling as described previously produces high-frequency decoupling, whereas lower frequency response coupling is suppressed by feedback. More complex dynamic compensators might be formulated, but this would increase the order of the control system and does not appear to be required in this application.

The singular values of the system with the inner loops closed and the feedforward gain matrix in place, $G_{\text{closed loop}}$, is shown in Fig. 2. The singular values are essentially the same for frequencies greater than 0.01 rad/s and exhibit the characteristics of first-order systems with a bandwidth of 4 rad/s. At frequencies lower than 0.01 rad/s, the singular values are not flat since we do not have complete cancellation of low-frequency poles and zeros.

Figure 3 shows the singular value plot of the identity matrix minus the transfer function matrix of $G_{\text{closed loop}}$. For a system to have good decoupling characteristics, the $\bar{\sigma}[I - G_{\text{closed loop}}]$ must be smaller than unity in the frequency range of interest. As shown in Fig. 3, decoupling of the heave mode is not achieved at low frequencies, but the controller has been able to achieve small gains over the frequency range of interest, 0.1–10 rad/s. The coupling at low frequencies results because of incomplete pole/zero cancellation.

Outer-Loop Design

Once the inner-loop system has been designed to approximate the desired transfer functions given in Eqs. (1–6), various command response types can be achieved by simple single-loop feedback or feedforward loops. The design of an ACAH system will be used to illustrate the method. With the inner feedback loops closed, the helicopter is effectively decoupled into four first-order systems. The approximate transfer function between the roll rate and roll rate command is shown in Fig. 4, and various response types can be achieved by simple SISO compensators as shown.

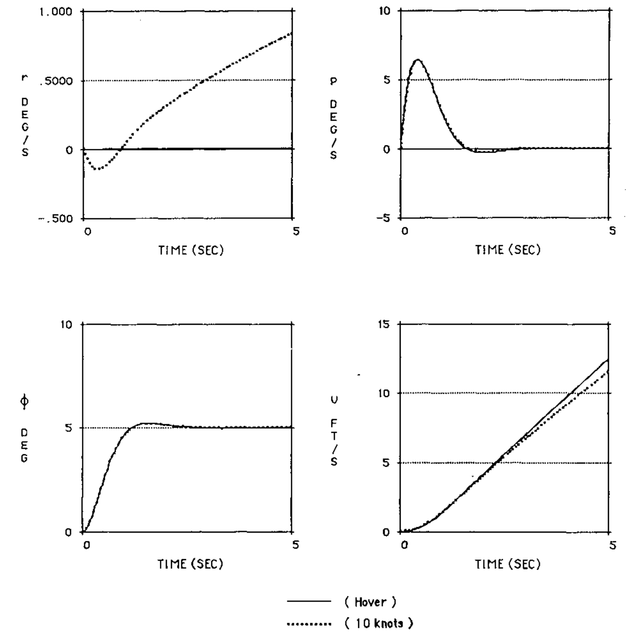

Fig. 5 Lateral and directional responses to a 5-deg step roll attitude command, eighth-order design model at hover and 10 kt.

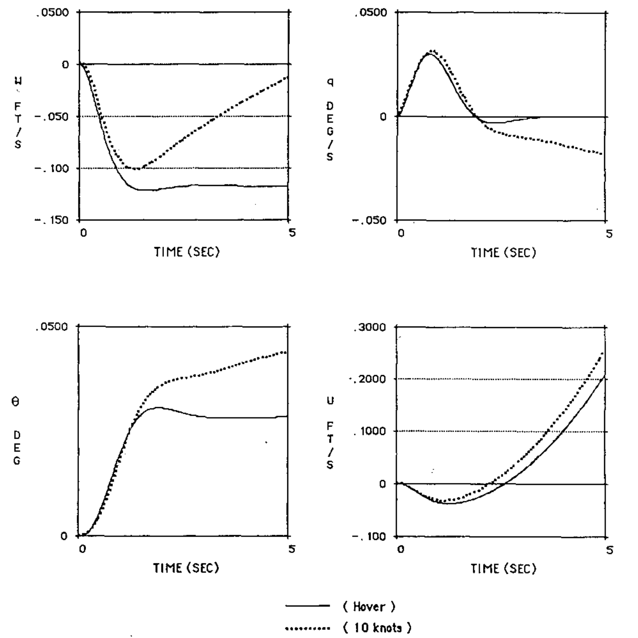

Fig. 6 Longitudinal responses to a 5-deg step roll attitude command, eighth-order design model at hover and 10 kt.

For ACAH, the lateral and longitudinal angles ϕ and θ can be fed back by outer loops with a proportional gains of K_ϕ and K_θ. Thus, only the roll and pitch angle measurements are needed for the outer-loop design. The resulting transfer functions between commanded and actual pitch and roll angle are given in Eqs. (3) and (14). Both K_ϕ and K_θ were selected as 2. The transfer functions are second order with a natural frequency $\omega_{n\phi,\theta} = 2.83$ rad/s and a damping ratio $\zeta = 0.707$:

$$\frac{\phi}{\phi_c} = \frac{K_\phi \lambda_p}{(s^2 + \lambda_p s + K_\phi \lambda_p)} \tag{13}$$

$$\frac{\theta}{\theta_c} = \frac{K_\theta \lambda_q}{(s^2 + \lambda_q s + K_\theta \lambda_q)} \tag{14}$$

With the outer loops closed, the eigenvalues are shown in Table 2. Other response types can be achieved by suitable selection of $K(s)$.

Evaluation of Performance

The properties of the control system that must be evaluated are 1) the nominal performance, how well the control system meets the performance specifications when applied to the model used for control law design; 2) stability robustness, the ability of the control system to maintain stability despite the presence of uncertainties in the mathematical model used for design; and 3) robust performance, the ability to meet performance specifications in the presence of model uncertainties. It is necessary to assess the stability robustness of this system with respect to modeling errors and parameter variations. Nominal performance for these control laws is easily verified from the singular value plots in Figs. 2 and 3 and from the time history plots such as those given in Figs. 5 and 6. These figures show the response of the nominal eighth-order helicopter model to a 5-deg commanded roll input. The roll rate and roll angle response are typical of a second-order system. The side velocity increases essentially linearly with time, as would be expected. The longitudinal and directional responses are very small, indicating that decoupling has been achieved. Similar decoupled responses were achieved for commands about the other axes.

Stability robustness is evaluated using unstructured singular values. An error model that gives an approximation of the modeling uncertainties is developed. The error model used in this paper approximates the unmodeled high-frequency dynamics of structural modes, actuators, filters, sensors, sampling delays, and computational delays associated with digital implementation of the flight control system with effective time delays.[20] This model is as rigorous as the more complex error model described in Ref. 11 and is much simpler to use. The total effective time delay for each of the four input channels is assumed to be the same. Based on the data in Ref. 22, a compilation of the various delays is given in Table 3. The total effective time delay is chosen to be 0.15 s.

The multiplicative (relative) error model given by Eq. (15) is utilized in the stability robustness test. The multiplicative error matrix is defined as

$$G_{\text{true}}(s) = G(s)\left[I + E(s)\right] \tag{15}$$

A sufficient condition[21] for the closed-loop system to be robustly stable with respect to $E(s)$ is

$$\underline{\sigma}\left(I + \left[K(s)G(s)\right]^{-1}\right) > \bar{\sigma}\left[E(s)\right] \quad \text{for all} \quad s = j\omega \tag{16}$$

where $0 < \omega < \infty$.

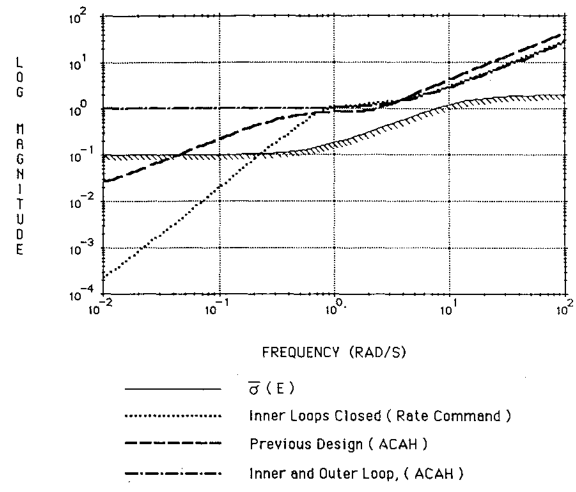

Fig. 7 Unstructured singular value robustness tests.

Table 4 Maximum actuator and actuator rate deflections

Maneuver	Maximum actuator deflection, deg	Maximum actuator rate, deg/s
5 ft/s vertical velocity	3.4 directional	37.2 directional
5 deg roll attitude	0.88 lateral	9.13 lateral
5 deg pitch attitude	4.33 longitudinal	45.7 longitudinal
5 deg/s yaw rate	1.65 directional	18.2 directional

Using a first-order Pade approximation for the total effective time delay and assuming a 10% error at low frequencies to approximate errors in the aerodynamic coefficients, the error model is given as

$$E(s) = -\left[\frac{2\Delta\tau s + 0.2}{\Delta\tau s + 2}\right] I \tag{17}$$

From Eq. (17), the modeling error is estimated to lie within the shaded region shown in Fig. 7. If the modeling error is indeed within this region, stability of the closed-loop system is assured if Eq. (16) is satisfied.

Figure 7 shows $\underline{\sigma}(I + [K(s)G(s)]^{-1})$ for the inner loop and inner plus outer loop control laws developed in this paper and for the control law developed by the authors in Ref. 11. It can be seen that all control laws give good stability robustness at high frequencies; however, at lower frequencies the control law of Ref. 11 indicates the possibility of instabilities at low frequencies. This was verified by computer simulations. With the inner loop only closed, the control law developed in this study also indicates the possibility of low-frequency instabilities. This was verified by a stochastic root locus analysis and simulations.[23] Figure 7 illustrates that the low-frequency stability robustness of the helicopter is substantially improved when the outer loop is closed. A measure of the robustness of a MIMO system is the smallest difference between the minimum singular value of $[I + K(s)G(s)^{-1}]$ and the maximum singular value of the multiplicative error $E(s)$. This can be regarded as a MIMO gain margin. As shown in Fig. 7, the minimum difference between the two curves is 6.7 dB at about 7.39 rad/s.

The effects of parameter variations on the response are illustrated in Figs. 5 and 6, where the response of the helicopter at a forward velocity of 10 kt is determined using the control gains calculated at hover. It can be seen that the lateral responses are practically identical for the hover and 10 kt flight conditions. There is more coupling between the longitudinal and directional modes at the 10 kt condition than at hover; however, the magnitudes of the longitudinal and directional responses are still small. Of course, the control laws could be gain scheduled with forward velocity to improve performance.

A higher order simulation was developed to examine the effects of rotor, actuator, and time delays on the response of the controlled helicopter. The 12th-order model of the helicopter, which included second-order models of the rotor tip path plane advancing and regressing modes, was used. The actuator dynamics were modeled as second-order systems:

$$\frac{\delta}{\delta_c} = \frac{30^2}{s^2 + 2(0.707)30s + 30^2} \tag{18}$$

From Table 3, the time delay due to filtering, sampling, and computation was 0.05 s. This was modeled by a first-order Pade approximation:

$$e^{-0.5s} = \frac{40 - s}{s + 40} \tag{19}$$

The inclusion of rotor dynamics, actuator dynamics, and time delays results in a 24th-order evaluation model. The response of the design model and the evaluation model to a 5-deg step roll input are compared in Figs. 8 and 9. The higher order model exhibits more oscillatory transient response than does the design model, but the steady-state responses are almost identical. ACAH response is maintained, and decoupling is still very good. Responses to other inputs were equally good. The actuator deflections and deflection rates for the 5-deg commanded roll angle are shown in Figs. 10 and 11. Deflections were small compared to representative maximum values of 9 deg for collective, 8.75 deg for lateral, 15 deg for longitudinal, and 18.5 deg for directional. Deflection rates for this maneuver were larger but still less than representative maximum values of 22 deg/s for collective, 21 deg/s for lateral, 36 deg/s for longitudinal, and 59 deg/s for directional.

Table 4 illustrates maximum deflection and deflection rates for step commands about each axis. All deflections were well within limits, as were rates, except for a 5-deg pitch angle

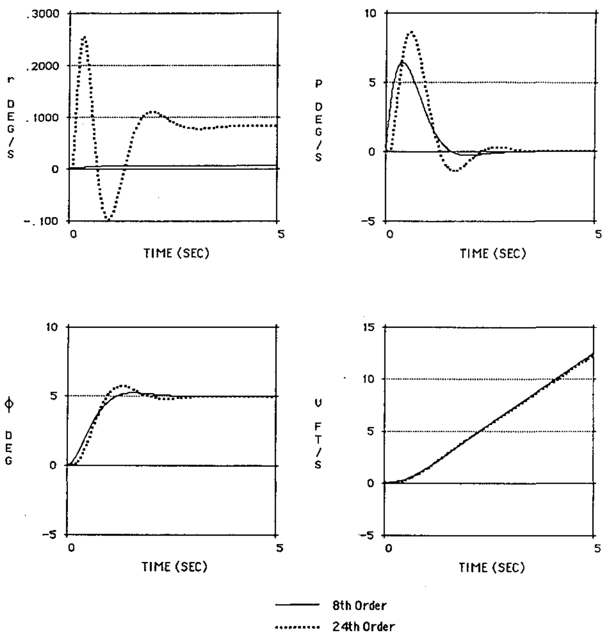

Fig. 8 Lateral and directional responses of 8th- and 24th-order models at hover to a 5-deg step roll attitude command.

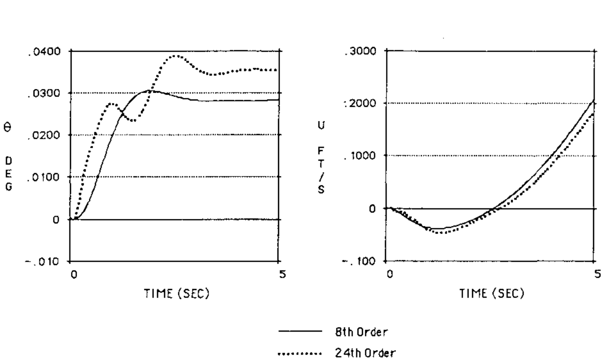

Fig. 9 Longitudinal responses of 8th- and 24th-order models at hover to a 5-deg step roll attitude command.

command, which resulted in a deflection rate above the maximum value. This would result in an effective reduction in gain in the longitudinal channel, which would result in reduced bandwidth. Since actual commands are never as severe as a step input, deflection rates might not saturate in practice.

The Bode diagrams for the closed-loop roll axis response for the 24th-order model are given in Figs. 12 and 13. The phase margin limited bandwidth is the frequency at which the phase is −135 deg (phase margin of 45 deg). From Fig. 13 the bandwidth is 3.6 rad/s. Pilots are sensitive to the shape of the phase curve at frequencies above the bandwidth frequency, and the phase delay parameter τ_p, as defined in Eq. (20), is used in

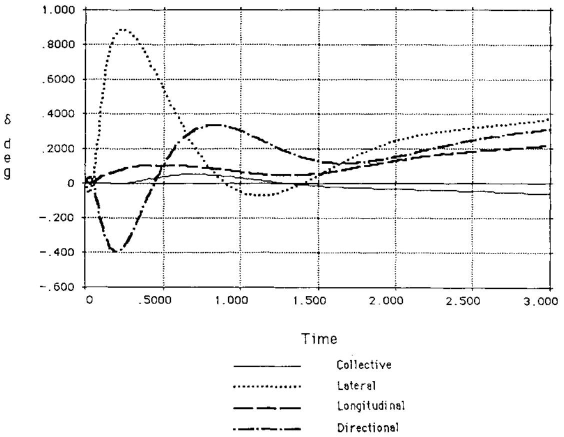

Fig. 10 Actuator deflections for a 5-deg step roll attitude command.

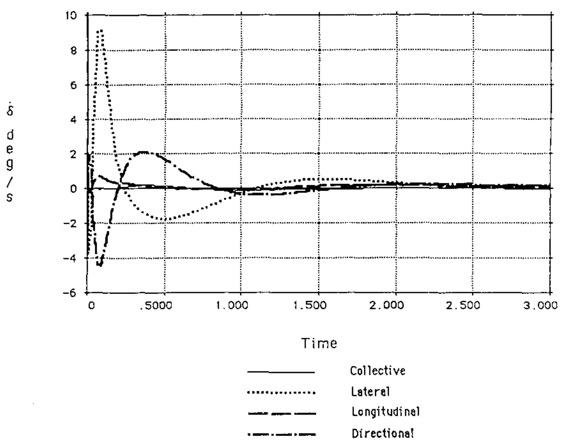

Fig. 11 Actuator deflection rates for a 5-deg step roll attitude command.

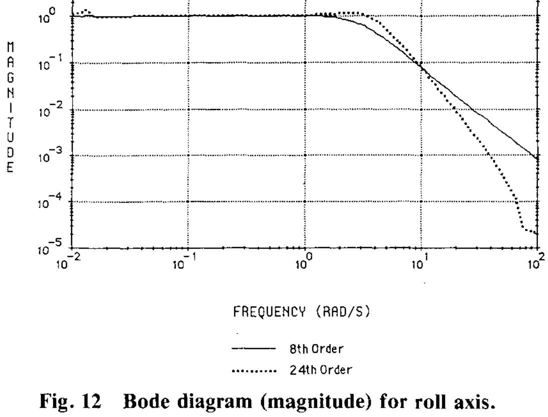

Fig. 12 Bode diagram (magnitude) for roll axis.

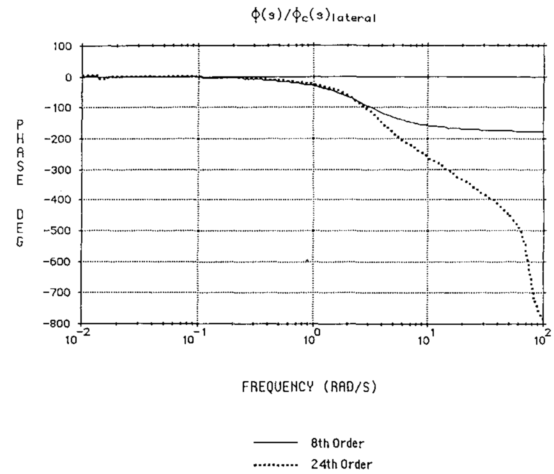

Fig. 13 Bode diagram (phase angle) for roll axis.

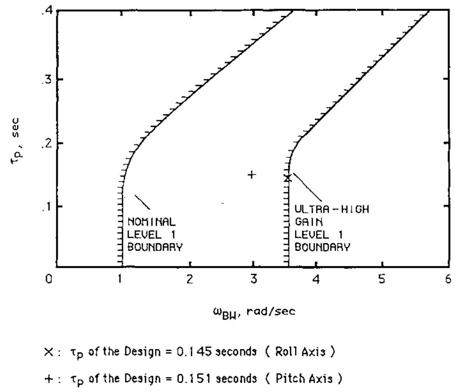

Fig. 14 Handling qualities for roll and pitch axes.

conjunction with the bandwidth for the evaluation of the handling qualities[2]:

$$\tau_p = \frac{\Delta\Phi(2\omega_{180})}{57.3(2\omega_{180})} \tag{20}$$

From Fig. 13, $\Delta\Phi 2\omega_{180}$ is determined to be 79.6 deg, and $2\omega_{180}$ is determined to be 9.6 rad/s. The phase delay of the overall system is calculated to be $\tau_p = 0.145$ s. As shown in Fig. 14, this value is on the boundary between ultrahigh-gain level 1 handling quality requirements and nominal level 1 quality requirements. Similar analysis for pitch yields a bandwidth of 3 rad/s and a phase delay of 0.151 s. This places the pitch response in range for nominal level 1 handling qualities. However, rate saturation might reduce the bandwidth, thereby degrading handling qualities.

Conclusions

The methodology described in this paper provides a straightforward approach to the design of helicopter flight control systems that provide response types necessary to met the new handling quality specifications. When applied to the nominal design model, the control laws provide excellent command response tracking, modal decoupling, and handling qualities as specified by the new requirements. Stability and performance robustness are good; however, some commands may result in actuator rate saturation, which may degrade handling qualities. Control laws proposed in the paper are of relatively low order, and implementation should be simple. Further application of the method to more realistic mathematical models appears to be warranted.

Appendix

A_{12} =

Columns 1–6:

−0.0232	0.0055	−0.0059	−0.0521	0.7174	0.0211
−0.0569	−0.0557	−0.0061	−0.8480	−0.0519	0.7144
−0.0772	−0.0748	−0.3803	−0.0413	−0.3013	2.0310
−0.0219	−0.0339	−0.0039	−0.5980	−0.0431	0.4003
0.0039	−0.0012	−0.0036	0.0077	−0.1631	0.0029
0.0293	0.0161	−0.0007	0.1352	0.0423	−0.4930
0.0000	0.0000	0.0000	1.0000	0.0005	−0.0154
0.0000	0.0000	0.0000	0.0000	0.9994	0.0343
1.1291	−0.3616	0.0036	−70.2077	−34.8969	0.3893
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−0.3414	−1.0981	0.0058	−34.5620	70.2831	0.4193
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Columns 7–12:

0.000	−32.1662	0.5483	−5.0970	−0.0860	9.7470
32.1473	−0.0169	−0.0881	9.7240	−0.5523	5.0490
1.1024	0.4938	−0.0078	−0.9641	0.0288	−1.0320
0.0000	0.0000	−0.0687	7.9190	−0.4014	32.8146
0.0000	0.0000	−0.0703	5.8210	0.0111	−1.2540
0.0000	0.0000	0.0377	−5.0972	0.0573	−3.4304
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	−27.5297	−511.8210	−67.8411	−941.9460
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	67.9287	935.7810	−27.2686	−540.1146
0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

Dimensional 12th-order helicopter A matrix

B_{12} =

−4.9320	−8.8350	25.4900	0.0004
−3.9280	25.5100	8.7880	18.7500
−334.6000	0.6213	−0.2959	0.0133
−5.3705	18.5905	6.4560	10.0067
−0.6877	1.1350	−3.2690	−0.1231
12.8487	−2.5707	−1.0964	−11.9561
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000
−1.7003	855.9650	−395.4310	0.1293
0.0000	0.0000	0.0000	0.0000
4.1285	380.5095	849.5440	−9.9949
0.0000	0.0000	0.0000	0.0000

Dimensional 12th-order helicopter B matrix

C_{12} =

0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0

Dimensional 12th-order helicopter C matrix

A_8 =

−0.0199	−0.0058	−0.0058	−0.7304	1.1197	0.0268	0.0000	−32.1662
−0.0452	−0.0526	−0.0061	−1.2567	−0.7517	0.7154	32.1473	−0.0169
−0.0788	−0.0747	−0.3803	0.0375	−0.2334	2.0306	1.1024	0.4938
0.0094	−0.0536	−0.0037	−2.9979	−0.5308	0.4155	0.0000	0.0000
0.0076	0.0040	−0.0036	0.0710	−0.5943	0.0013	0.0000	0.0000
0.0226	0.0151	−0.0007	0.4058	0.4069	−0.4940	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0005	−0.0154	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.9994	0.0343	0.0000	0.0000

Dimensional eighth-order design model A matrix

$B_8 =$

−4.9064	−0.9103	30.4980	−0.0834
−3.9661	30.7245	0.5566	18.8079
−334.5965	−0.3527	0.4905	0.0099
−5.3885	47.5204	1.3582	9.9301
−0.7124	0.5789	−8.4358	−0.0700
12.8683	−5.9781	3.1746	−11.9831
0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000

Dimensional eighth-order design model B matrix

$C_8 =$

0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0

Dimensional eighth-order design model C matrix

$K =$

0.0002	0.0002	−0.0108	−0.0041	0.0028	−0.0060	−0.0193	0.0094
0.0007	−0.0009	0.0011	0.0322	0.0333	0.0780	0.0012	0.0016
−0.0009	−0.0006	0.0015	−0.0041	−0.4016	0.0084	0.0072	−0.0055
−0.0022	−0.0007	−0.0117	−0.0554	−0.1540	−0.3357	−0.0190	0.0079

Dimensional regulator feedback gain matrix

$H =$

−1.2824	−0.0007	−0.0021	0.0005
0.1468	0.2130	0.0586	0.0832
0.1286	0.0154	−0.6223	0.0090
−1.4155	−0.1032	−0.1963	−0.3990

Dimensional feedforward gain matrix

$V_d =$

u	1.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
v	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
w	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
p	0.0000	0.0000	0.0000	0.0000	0.0000	0.9701	0.0000	0.0000
q	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.9701	0.0000
r	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
ϕ	0.0000	0.0013	0.0000	0.0000	0.0000	−0.2425	0.0000	0.0000
θ	0.0000	0.0000	0.0000	0.0005	0.0000	0.0000	−0.2425	0.0000

Desired nondimensional closed-loop eigenvectors

$V_a =$

u	0.9992	−0.0007	−0.0006	0.9993	0.0260	0.0025	−0.1063	0.0000
v	−0.0006	0.9998	0.9998	−0.0007	−0.0339	0.0467	−0.0077	−0.0244
w	−0.0010	−0.0002	0.0001	−0.0010	0.9991	0.0015	0.0025	−0.0008
p	−0.0001	0.0000	0.0000	0.0000	0.0015	0.9691	0.0006	0.0020
q	0.0000	0.0000	−0.0001	0.0000	0.0025	0.0006	0.9646	−0.0022
r	0.0000	0.0000	0.0000	0.0000	−0.0008	0.0011	−0.0002	0.9997
ϕ	0.0360	0.0138	0.0061	0.0360	−0.0004	−0.243	−0.0003	0.0033
θ	0.0148	0.0168	0.0168	0.0120	−0.0006	−0.0002	−0.2410	−0.0080

Achievable nondimensional closed-loop eigenvectors (see Ref. 11 for nondimensionalization).

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